Hydromagnetic Oscillations in a Conducting Medium with Hall Couduct-ivity under the Uniform Magnetic Field

著者	Kato Yoshio, Tsutomu Tamao
雑誌名	Science reports of the Tohoku University. Ser. 5, Geophysics
巻	7
号	3
ページ	147-164
発行年	1956-03
URL	http://hdl.handle.net/10097/44544

Hydromagnetic Oscillations in a Conducting Medium with Hall Conductivity under the Uniform Magnetic Field

By YOSHIO KATO and TSUTOMU TAMAO
Geophysical Institute, Faculty of Science, Tohoku University,

(Received 25 December 1955)

Abstract

The hydromagnetic oscillations in the case of periodic and aperiodic disturbances in a conducting medium in which the Hall conductivity is larger than direct one are discussed

Under such conditions, the poloidal and the toroidal type fields, will couple each other and consequently damped oscillations will occur independtly to the boundary conditions.

1 Introduction

It is obvious that the terrestrial magnetic disturbances, such as SD or others are accompanied by ionospheric disturbances, but up to present there is a little investigation on the cause of magnetic rapid pulsations.

It may be possible to separate the observed magnetic disturbances in two parts, the one is caused by an origin of distrubances consist in the ionosphere itself and the other is caused by those as the secondary effects of extraterrestrial ones in the other. Concerning to the later, the shielding effect of the ionosphere was disscused by many authors, we must always consider this effect to interpret the phenomena [1, 2, 3]. In the above discussions, however, the ionosphere is assumed as a solid conductor and therefore the skin effect is overestimated more than in fluid [4].

The disturbances in a conducting gas must be studied by considering individual ion motions at first and then sum up these effects, but in general, this method is almost impossible practically except in the special cases [5, 6]. However, the problem is simplified by taking the macroscopic quantity (e., g., equations of motion in contineous medium and conductivity as a mean value.). From this standpoint, recently we investigated the pulsations by connecting it to magneto-hydrodynamic oscillations in the ionosphere [7, 8, 9].

In this paper, we have some discussions of hydromagnetic oscillations in a conducting fluid having non-isotropic conductivity under the simple model. When the Hall conductivity is larger than diret one, Cowling [10] pointed out firstly that there is damping motions with the period of $2\pi\rho/\sigma_2 H^2{}_0$ from the hydrodynamic equations of motion in the two dimensional case. We obtained those damped oscillations as a particular solution of differential equation and transient solution in the aperiodic case. It is interesting in connection with the damped pulsations accompanying the s.c. of magnetic storm that the oscillation of this kind may be always occured as a transient phenomenon, for instance, the sudden change of motions of fluid, sudden appearance of electric or magnetic fields and sudden increase or decrease in electic current.

Moreover, as Roberts [4] discussed, the skin depth is much larger in fluid than in solid. Of course, there is the "streak effect" which investigated by Walén [11] and Roberts [4].

In general, the phase velocity and the attenuation factor in propagation of disturbances along the magnetic line of force depend upon lateral dimension of variation and there are smaller velocity and larger attenuation when this dimension becomes smaller. Those above stated is valid when the rate of time change is not much rapid. Above discussions here is in the case of vertical permanent magnetic field. For the oblique field, the method is similar but it cannot avoid that the notations become complexity.

2 Field equations and elementary solutions

We consider a conductiong fluid having non-isotropical conductivity under the uniform magnetic field. Take the right-handed Cartesian coordinate and for convenience direction of permanent magentic field coincides with z-axis.

We neglect the displacement current in the conductor with compared to conduction current, then Maxwell's equations become

$$\mathrm{curl}\, \boldsymbol{H} = 4\pi \boldsymbol{j} \tag{2.1}$$

$$\mathrm{curl}\, \boldsymbol{E} = -\frac{\partial \boldsymbol{H}}{\partial t} \tag{2.2}$$

The transport equation is

$$\boldsymbol{j} = \sigma_0 \boldsymbol{E}'_{\parallel} + \sigma_1 \boldsymbol{E}'_{\perp} + \frac{\sigma_2}{H} (\boldsymbol{H} \times \boldsymbol{E}'_{\perp}) \tag{2.3}$$

where

$$\boldsymbol{E}' = \boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{H}) \tag{2.4}$$

σ_0, σ_1 and σ_2 are parallel, direct transverse and Hall conductivity respectively.

$$\mathrm{div}\, \boldsymbol{H} = 0 \tag{2.5}$$

If the fluid conductor is incompressible and force acting to conductor are electromagnetic force and pressure only, and moreover if the stationary motion is not regard absent, then equation of motion

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho (\boldsymbol{v} \cdot \mathrm{grad})\, \boldsymbol{v} = \boldsymbol{j} \times \boldsymbol{H} - \mathrm{grad}\, \theta \tag{2.6}$$

where θ is pressure, ρ is density of conducting fluid.

$$\operatorname{div} \boldsymbol{\nu} = 0 \tag{2.7}$$

When the motion of disturbances is small, then the inertia term in the left side of eq. (2.6) can be neglected.

When the current $\boldsymbol{j}$ is solenoidal, Piddington [12] has introduced following equations eliminating the $\boldsymbol{j}$ and $\boldsymbol{E}$ from eq. (2.1) to (2.5)

$$\nabla^2 H_x - \frac{\sigma_2}{\sigma_1} \frac{\partial}{\partial z} \operatorname{curl}_x \boldsymbol{H} = 4\pi\sigma_3 \left\{ \frac{\partial H_x}{\partial t} - \operatorname{curl}_x (\boldsymbol{\nu} \times \boldsymbol{H}) \right\}$$
$$- \left(1 - \frac{\sigma_3}{\sigma_0}\right) \frac{\partial}{\partial y} \operatorname{curl}_z \boldsymbol{H} \tag{2.8}$$

$$\nabla^2 H_y - \frac{\sigma_2}{\sigma_1} \frac{\partial}{\partial z} \operatorname{curl}_y \boldsymbol{H} = 4\pi\sigma_3 \left\{ \frac{\partial H_y}{\partial t} - \operatorname{curl}_y (\boldsymbol{\nu} \times \boldsymbol{H}) \right\}$$
$$+ \left(1 - \frac{\sigma_3}{\sigma_0}\right) \frac{\partial}{\partial x} \operatorname{curl}_z \boldsymbol{H} \tag{2.9}$$

$$\nabla^2 H_z - \frac{\sigma_2}{\sigma_1} \frac{\partial}{\partial z} \operatorname{curl}_z \boldsymbol{H} = 4\pi\sigma_3 \left\{ \frac{\partial H_z}{\partial t} - \operatorname{curl}_z (\boldsymbol{\nu} \times \boldsymbol{H}) \right\} \tag{2.10}$$

where

$$\sigma_3 = \sigma_1 + \frac{\sigma_2^2}{\sigma_1} \tag{2.11}$$

is the apparent conductivity in the direction of electric force ($\boldsymbol{\nu}\times\boldsymbol{H}$) provided that an electric field prevents the flow of the Hall current.

Equations (2.8), (2.9), and (2.10) are different from corresponding equations for scalar conductivity in respect to additional terms, that is, space derivatives of the component of curl $\boldsymbol{H}$.

Due to these space derivatives, it becomes more difficult to solve the eq. (2.8), (2.9) and (2.10) with compared to the case of scalar conductivity. Eq. (2.8), (2.9) and (2.10), however, are simplified in some kind gases. In the binary gas, σ_3 equals to σ_0 and the terms involving $1-\sigma_3/\sigma_0$ in the right-handside of eq. (2.8) and (2.9) may be eliminated. Also when the space derivative of $\operatorname{curl}_z \boldsymbol{H}$ can be neglected, the same terms are eliminated. Moreover, $\partial/\partial z=0$, that is, in the problem such that the disturbance propagate to across the magnetic field, the term of σ_2/σ_1 is eliminated. Denote the permanent magnetic field by $\boldsymbol{H}_0$ (o, o, H_0) and disturbing magnetic field by $\boldsymbol{h}$ (h_x, h_y, h_z,),
then

$$\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{h} \tag{2.12}$$

Assuming $\boldsymbol{H}_0 \gg \boldsymbol{h}$, the terms of square and product of h and v are neglected.
From eq. (2.1) and (2.6)

$$\rho \frac{\partial \boldsymbol{\nu}}{\partial t} = \frac{1}{4\pi} (\operatorname{curl} \boldsymbol{H} \times \boldsymbol{H}) - \operatorname{grad} \theta \tag{2.16}$$

Operateing div to the both sides of eq (2.6) and taking into account that $\boldsymbol{H}$ and $\boldsymbol{\nu}$ are solenoidal, then

$$\nabla^2\theta = -(4\pi)^{-1} (\boldsymbol{H}\cdot\nabla^2 \boldsymbol{H}) = -(4\pi)^{-1} (\boldsymbol{H}\cdot\nabla^2 \boldsymbol{h}) \tag{2.13}$$

Consider the following poloidal and toroidal type magnetic fields

$$\boldsymbol{h}^{(s)} = \begin{cases} \dfrac{\partial P}{\partial x}\dfrac{\partial S}{\partial z} \\ \dfrac{\partial P}{\partial y}\dfrac{\partial S}{\partial z} \\ \lambda^2 PS \end{cases} \quad (2.14)\,, \qquad \boldsymbol{h}^{(t)} = \begin{cases} \dfrac{\partial P}{\partial y} T \\ -\dfrac{\partial P}{\partial x} T \\ 0 \end{cases} \quad (2.15).$$

These expressions have been introduced by Price [13] and Gordon [14], (see [15]) in the case of soild electromagnetic induction.

Where S (z, t) and T (z, t) are functions of z and t alone and P (x, y) is function of x and y which satisfies the folloing equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \lambda^2 P = 0 \tag{2.16}$$

where λ is any positive number, and this function must be determined by the source distribution of fields.

Since the function of this type which changes in lateral directions is necessarily introduced in the solution of dielectric in the quasi-stationary state, in order to satisfy the boundary conditions, this function must be similarly introduced in the conducting medium. Then, $\boldsymbol{j}$, $\boldsymbol{E}$, θ, and $\boldsymbol{v}$ become following equations from the above equations respectively

$$4\pi \boldsymbol{j}^{(s)} = \begin{cases} -\dfrac{\partial P}{\partial y}\left(\dfrac{\partial^2 S}{\partial z^2} - \lambda^2 S\right) \\ \dfrac{\partial P}{\partial x}\left(\dfrac{\partial^2 S}{\partial z^2} - \lambda^2 S\right) \\ 0 \end{cases} \quad (2.17)\,, \qquad 4\pi \boldsymbol{j}^{(t)} = \begin{cases} \dfrac{\partial P}{\partial x}\dfrac{\partial T}{\partial z} \\ \dfrac{\partial P}{\partial y}\dfrac{\partial T}{\partial z} \\ \lambda^2 PT \end{cases} \quad (2.18)$$

$$\boldsymbol{E}^{(s)} = \begin{cases} -\dfrac{\partial P}{\partial y}\dfrac{\partial S}{\partial t} \\ \dfrac{\partial P}{\partial x}\dfrac{\partial S}{\partial t} \\ 0 \end{cases} \quad (2.19)\,, \qquad \boldsymbol{E}^{(t)} = \begin{cases} \dfrac{\partial P}{\partial x} T' \\ \dfrac{\partial P}{\partial y} T' \\ \dfrac{\lambda^2}{4\pi\sigma_0} PT \end{cases} \quad (2.20)$$

where

$$\frac{\partial T'}{\partial z} = \frac{\partial T}{\partial t} + \frac{\lambda^2}{4\pi\sigma_0} T \tag{2.21}$$

$$\theta^{(s)} = -(4\pi)^{-1}\lambda^2 H_0 PS \quad (2.22)\,, \qquad \theta^{(t)} = 0 \quad (2.23)$$

$$\rho\frac{\partial \boldsymbol{v}^{(s)}}{\partial t} = \begin{cases} \dfrac{H_0}{4\pi}\dfrac{\partial P}{\partial x}\dfrac{\partial^2 S}{\partial z^2} \\ \dfrac{H_0}{4\pi}\dfrac{\partial P}{\partial y}\dfrac{\partial^2 S}{\partial z^2} \\ \lambda^2\dfrac{H_0}{4\pi} P\dfrac{\partial S}{\partial z} \end{cases} \quad (2.24)\,, \qquad \rho\frac{\partial \boldsymbol{v}^{(t)}}{\partial t} = \begin{cases} \dfrac{H_0}{4\pi}\dfrac{\partial P}{\partial y}\dfrac{\partial T}{\partial z} \\ -\dfrac{H_0}{4\pi}\dfrac{\partial P}{\partial x}\dfrac{\partial T}{\partial z} \\ 0 \end{cases} \quad (2.25).$$

The differential equation which are satisfied by S and T may be obtained from to sabstitute the above given expressions for $\boldsymbol{h}$ and $\boldsymbol{v}$ to eq, (2.8), (2.9) and (2.10). Neither poloidal nor toroidal type fields satisfy these diff. equations, but these diff. equations may be satisfied with the fields as a sum of poloidal and toroidal type fields. This circumstance require a coupling of two types. It is interesting that there is a coupling in such a simple model. A coupling is absent in the case of scalar conductivity and even the case of $\sigma_2 \ll \sigma_1$ and poloidal and toroidal fields may be present as solutions independently each other. If the disturbance in the ionosphere are considered, above circumstance are important. In the E-layer, σ_2/σ_1 is about 10, while $\sigma_2 \ll \sigma_1$ in the F layer. In this characteristics two regions are quitely different, that is, in the E–layer coupling cannot be neglected, on the other hand, F-layer does not so and may has poloidal and toroidal fields independently.

Let $\boldsymbol{h}=\boldsymbol{h}^{(s)}+\boldsymbol{h}^{(t)}$ and $\boldsymbol{v}=\boldsymbol{v}^{(s)}+\boldsymbol{v}^{(t)}$ and substitute these expressionas to diff. eq., then, from eq. (2.10) and (2.16)

$$\frac{\sigma_2}{\sigma_1}\frac{\partial^2 T}{\partial t\partial z}=\frac{\partial}{\partial t}\left(\frac{\partial^2 S}{\partial z^2}-\lambda^2 S\right)-4\pi\sigma_3\frac{\partial^2 S}{\partial t^2}+\frac{\sigma_3 H_0^2}{\rho}\frac{\partial^2 S}{\partial z^2} \tag{2.26}$$

from eq. (2.8) and (2.16), we obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}-\lambda^2 T\right)+\frac{\sigma_2}{\sigma_1}\frac{\partial^2}{\partial t\partial z}\left(\frac{\partial^2 S}{\partial z^2}-\lambda^2 S\right)-4\pi\sigma_3\frac{\partial^2 T}{\partial t^2}$$
$$+\frac{\sigma_3 H_0^2}{\rho}\frac{\partial^2 T}{\partial z^2}+\left(1-\frac{\sigma_3}{\sigma_0}\right)\lambda^2\frac{\partial T}{\partial t}=0 \tag{2.27}$$

and the identical equation with (2.26). From eq. (2.9), two diff. eq. which are identical (2.26) and (2.27) are obtained.

Eliminate $T(z, t)$ from eq. (2.26) and (2.27), then diff. eq. satisfying $S(z, t)$ is

$$\left[\left(\frac{\partial}{\partial t}+\omega_0\right)^2+\sigma^2\frac{\partial^2}{\partial t^2}\right]\frac{\partial^4 S}{\partial z^4}-\left[\left(\frac{\partial}{\partial t}+\omega_0\right)\left\{8\pi\sigma_3\frac{\partial^2}{\partial t^2}+\left(1+\frac{\sigma_3}{\sigma_0}\right)\lambda^2\frac{\partial}{\partial t}\right\}\right.$$
$$\left.+\lambda^2\sigma^2\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2 S}{\partial z^2}+\left(4\pi\sigma_3\frac{\partial^2}{\partial t^2}+\frac{\sigma_3}{\sigma_0}\lambda^2\frac{\partial}{\partial t}\right)\left(4\pi\sigma_3\frac{\partial^2}{\partial t^2}\right.$$
$$\left.+\lambda^2\frac{\partial^2}{\partial t^2}\right)S=0 \tag{2.28}$$

For the binary gas, (2.28) becomes

$$\left[\left(\frac{\partial}{\partial t}+\omega_0\right)^2+\sigma^2\frac{\partial^2}{\partial t^2}\right]\frac{\partial^4 S}{\partial z^4}-\left[2\left(\frac{\partial}{\partial t}+\omega_0\right)\left\{4\pi\sigma_3\frac{\partial^2}{\partial t^2}+\lambda^2\frac{\partial}{\partial t}\right\}\right.$$
$$\left.+\lambda^2\sigma^2\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2 S}{\partial z^2}+\left(4\pi\sigma_3\frac{\partial^2}{\partial t^2}+\lambda^2\frac{\partial}{\partial t^2}\right)^2 S=0 \tag{2.28'}$$

When λ is equals to zero, eq. (2.28) reduces to

$$\left[\left(\frac{\partial}{\partial t}+\omega_0\right)^2+\sigma^2\frac{\partial^2}{\partial t^2}\right]\frac{\partial^4 S}{\partial z^4}-8\pi\sigma_3\left(\frac{\partial}{\partial t}+\omega_0\right)\frac{\partial^4 S}{\partial t^2\partial z^2}$$
$$+(4\pi\sigma_3)^2\frac{\partial^4 S}{\partial t^4}=0 \tag{2.29}$$

where

$$\omega_0=\frac{\sigma_3 H_0^2}{\rho} \qquad (2.30), \qquad\qquad \sigma=\frac{\sigma_2}{\sigma_1} \qquad (2.31)$$

3 Periodic oscillation

Differential equation (2.2 9) splits into two following equations

$$\frac{\partial^2 S_1}{\partial t^2}-\frac{1}{4\pi\sigma_3}(1-i\sigma)\frac{\partial^3 S_1}{\partial t\partial z^2}=V^2\frac{\partial^2 S_1}{\partial z^2} \qquad (3.1\text{A})$$

$$\frac{\partial^2 S_2}{\partial t^2}-\frac{1}{4\pi\sigma_3}(1+i\sigma)\frac{\partial^3 S_2}{\partial t\partial z^2}=V^2\frac{\partial^2 S_2}{\partial z^2} \qquad (3.1\text{B})$$

where

$$V^2=\frac{H_0^2}{4\pi\rho} \qquad (3.2).$$

V is the velocity of the Alfvén wave in the case of infinite scalar conductivity [16]. Considering a solution of the form $e^{i\omega t-kz}$, then from eq. (2.28), the dispersion equation becomes

$$2\{(1+\sigma^2)\omega^2-2i\omega_0\omega-\omega_0^2\}k^2_{\mp}=\left\{\left(\frac{\sigma_2}{\sigma_1}+\frac{\sigma_3}{\sigma_1}\right)\lambda^2+8\pi\sigma_3\omega_0\right\}\omega^2$$

$$-8\pi\sigma_3\omega^3 i+\lambda^2\omega_0\left(1+\frac{\sigma_3}{\sigma_0}\right)\omega_i\mp D \qquad (3.3)$$

where

$$D^2-(8\pi\sigma\sigma_3)^2\omega^6-\frac{16\pi}{\sigma_0}(\sigma\sigma_3)^2\lambda^2\omega^5 i+(16\pi_0\sigma_3\sigma^2\lambda^2+\sigma^4\lambda^4)\omega^4$$

$$-2\omega_0\sigma^2\left(1+\frac{\sigma_3}{\sigma_0}\right)\lambda^4\omega^3 i-\left(1-\frac{\sigma_3}{\sigma_0}\right)^2\omega_0^2\lambda^4\omega^2 \qquad (3.4).$$

We have two value of k considering the signs corresponding to sign $\mp$ in (3.3). and denote this to $k_{\mp}$. In the case of $\lambda=0$, these are expressed,

$$\begin{cases} k_+^2=-4\pi\sigma_3\omega^2/\{4\pi\sigma_3 V^2+\omega\sigma+i\omega\} \\ k_-^2=-4\pi\sigma_3\omega^2/\{4\mathrm{P}\sigma_3 V^2-\omega\sigma+i\omega\} \end{cases} \qquad (3.5)$$

corresponding to those obtained from eq. (3.3) and (3.4).

Therefore, there are in general two polarized waves.* It may be happened in the lightly ionized gas that the velocity and skin depth of these waves are very different when $\omega_0\leq\sigma\ \omega$, [6,17]. Phase velocity and skin depth are given by $\omega/I_m(k)$ and $1/Re\ (k)$ respectively.

$$\omega\ll 1, \qquad \begin{cases} V_\pm \rightarrow V=H_0/\surd(4\pi\rho) \\ z_\pm \rightarrow\infty\ (\omega^{-1/2}) \end{cases}$$

$$\omega\gg 1, \qquad \begin{cases} V_\pm \sim\infty\ (\omega^{1/2}) \\ z_+ \sim\sigma/\surd(\pi\sigma_3\omega), \quad \lambda+/z_+\sim 1/\surd(2\sigma^3)\ll 1 \\ z_- \sim\sigma/\surd(4\pi\sigma_3\omega), \quad \lambda_-/z_-\sim\surd(2\sigma^3)\gg 1 \end{cases}$$

* The case of $\omega_0=0$ (namely $\rho=\infty$) corresponds to solid induction.

where $\lambda_\pm$ are wave-length.

Also,

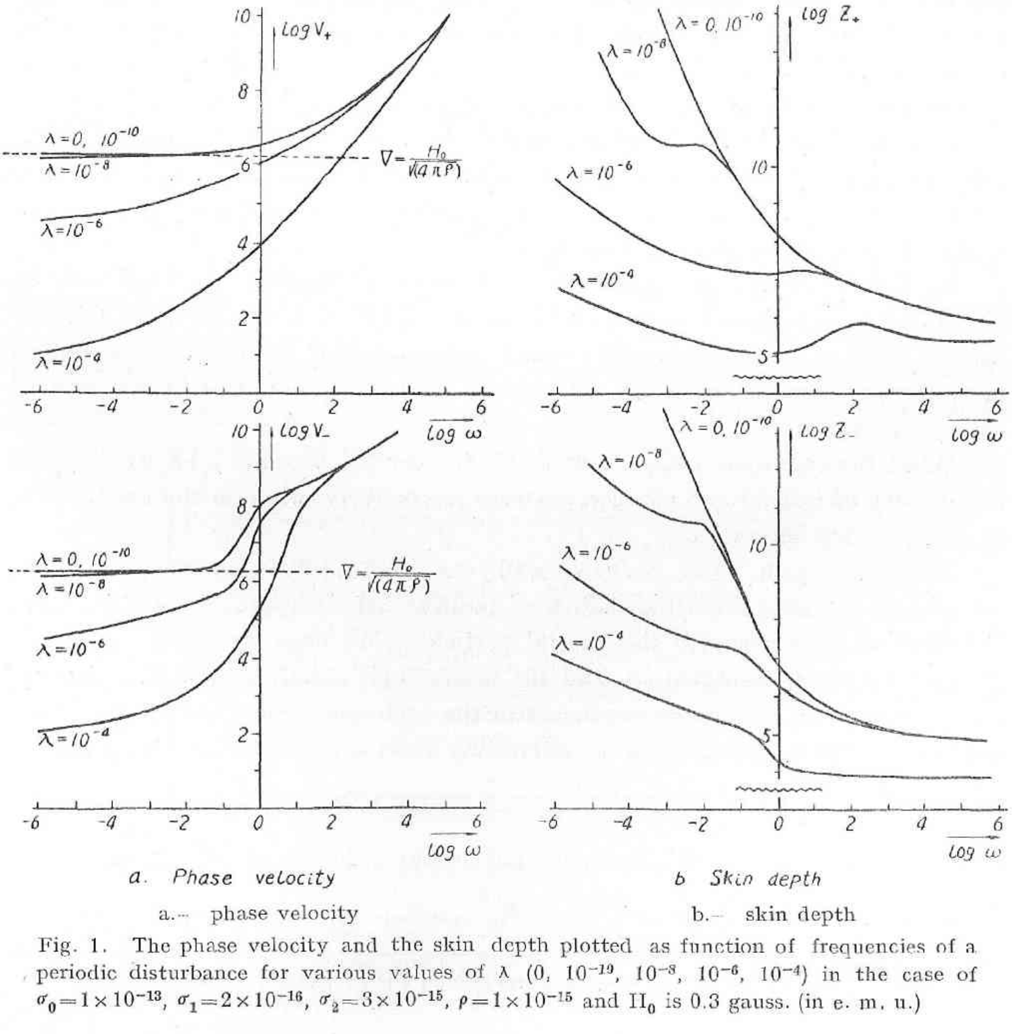
$$\frac{z_\pm}{z_0}=\left\{\frac{\left(1\pm\sigma\frac{\omega}{\omega_0}\right)^2+\left(\frac{\omega}{\omega_0}\right)^2}{\frac{\omega}{\omega_0}\left[\left\{\left(1\pm\sigma\frac{\omega}{\omega_0}\right)^2+\left(\frac{\omega}{\omega_0}\right)^2\right\}^{1/2}-\left(1\pm\sigma\frac{\omega}{\omega_0}\right)\right]}\right\}^{1/2}\begin{array}{l}>1 \text{ for all } \omega \ (\sigma\neq 0)\\ \geqq 1 \quad \omega\to\infty \ (\sigma=0)\end{array} \tag{3.6}$$

where

$$z_0=1/(2\pi\sigma_3\omega) \tag{3.7}$$

is the skin depth in the case of solid induction in the medium which has conductivity σ_3. Therefore, under the approximation of the conducting gas as a solid, this value is overestimated in general.

In Fig. I., phase velocity and skin depth are shown as a function of ω for various values of λ.



a.– phase velocity b.– skin depth

Fig. 1. The phase velocity and the skin depth plotted as function of frequencies of a periodic disturbance for various values of λ (0, 10^{-10}, 10^{-8}, 10^{-6}, 10^{-4}) in the case of $\sigma_0=1\times10^{-13}$, $\sigma_1=2\times10^{-16}$, $\sigma_2=3\times10^{-15}$, $\rho=1\times10^{-15}$ and H_0 is 0.3 gauss. (in e. m. u.)

From this figure, it is clear that velocity for $\lambda \neq 0$ are smaller than corresponding one for $\lambda=0$ and in the limitting case $\omega \gg 1$, velocity for the former tend to the later one. Similarly, skin depth for $\lambda \neq 0$ are smaller than for $\lambda=0$ and in the limit $\omega \gg 1$, both one tend to zero asymptotically.

For $\omega \ll 1$, every velocity tends to limiting value corresponding to the values of λ, especially, for $\lambda=0$, the velocity tends to Alfvén's one and there is no absorption.

From eq. (2.16), in general, $\lambda \sim L^{-1}$, where L is linear dimension of lateral distance in which fields varies appreciably. Therefore, a phenomenon of which variation in lateral direction is smaller, ,velocity becomes smaller and on the contrary absorption becomes stronger

Moreover, from Fig. I., it is shown that if the frequency becomes higher, phase velocity and attenuation become larger. This fact was first pointed out by Walén [11] (also Roberts [4]) and was called by the name of the "streak effect", that is, the higher frequency components travel faster but they decay more rapidly. He pointed out that the higher frequencies of initial disturbance will be a "streak" at first and soon decay. The main parts of the wave become increasingly composed of low frequencies, and since these are less dissipated, the wave motion will become increasingly pure in form.

In calculating the Fig.1., we assumed that ρ is order of 10^{-15}gr. cm^{-3}, but the value of (effectiv) density is, in general, a function of frequency. Furthermore, so far as we use a mean value as that of conductivity, wave frequency ω cannot be exceeded the gyrofrequency of ions (Ω_p).

In lightly ionized gas, the effect of background (neutral particles) may be introduced in equation of motion as the friction terms, consequently, the effective density as a macroscopic quantity which contributes to the disturbance depend upon the relations among ω, Ω_p, Ω_e and $\nu_{i,j}$ (collisional frequency of ith particle with jth particle).

When the condition $\omega \ll \nu_{i,j}$, is satisfied, the density becomes $\mu+\delta$, where μ and δ is density of neutral and charged particles respectively, while in the condition of $\Omega_p > \omega > \nu_{i,j}$, this becomes δ.

In the E region, there is $\Omega_p(\sim 2\times 10^2) < \nu_{p,n}(\sim 7\times 10^3)$ [6], consequently, the disturbance resulting from the motion of positive ions is appreciably subjected to the effect of collisions with the neutral particles. We have reexamine, therefore, the eq. of motion, transport eq. and the macroscopic quantities (effective density, current and force etc.) in connection with the collisional effect under the above conditions. These examination is interesting from a standpoint of the shielding effect of the ionospere.

4 Special damping oscillation as a particular solution of diff. equation

In eq. (2.28), let $S(z, t)_{being}\ e^{pt+kz}$, then

$$\Big[(p+\omega_0)^2+\sigma^2p^2\Big]k^4-\Big[(p+\omega_0)\Big\{8\pi\sigma_3 p^2+\Big(1+\frac{\sigma_3}{\sigma_0}\Big)\lambda^2p^2\Big\}+\lambda^2\sigma^2p^2\Big]k^2$$

$$+\left(4\pi\sigma_3 p+\frac{\sigma_3}{\sigma_0}\lambda^2\right)(4\pi\sigma_3 p+\lambda^2)\,p^2=0 \tag{4.1}$$

In the above equation, take the coefficient of k^4 equal to zero

$$(p+\omega_0)^2+\sigma^2p^2=(1+\sigma^2)\,p^2+2\omega_0 p+\omega_0^2 \tag{4.2}$$

or

$$p^2+2\frac{\sigma_1 H_0^2}{\rho}p+\frac{(\sigma_1^2+\sigma_2^2)H_0^4}{\rho^2}=0 \tag{4.2}$$

This equation is that of a damped oscillation. Solving this equation

$$p=-\alpha\pm i\beta \tag{4.3}$$

where

$$\alpha=\sigma_1 H_0^2/\rho \tag{4.4}$$

$$\beta=\sigma_2 H_0^2/\rho \tag{4.5}$$

that is, the solution of this equation shows a damping oscillation having the period of $2\pi/\beta$ and damping time of $1/\alpha$. If β/α $(=\sigma_2/\sigma_1=\sigma)$ $\gg 1$, this solution shows a periodic damping oscillation.

The values of k corresponding to this solution may be obtained to substitute eq. (4.3) in (4.1). For simplicity, take the case of $\lambda=0$, then

$$k^2{}_{\pm}=\frac{2\pi\sigma_3(-\alpha\pm i\beta)^2}{(\omega_0-\alpha)\pm i\beta} \tag{4.6}$$

In eq. (3.1A) and (3.1B), let $p=\alpha\pm i\beta$, then the same result may be obtained. Such damped oscillation was first discussed by Cowling [10] considering the equations of motion of two dimensional case in the conducting fluid having Hall conductivity and also investigated by Piddington [18] neglecting the contribution of current to the magnetic field and eq. of motion in lateral directional components. In the present case, the corresponding equations of motion for v_x and v_y is are

$$\begin{cases}\rho\dfrac{\partial v_x}{\partial t}+H_0^2\sigma_1 v_x-H_0^2\sigma_2 v_y=H_0\sigma_1E_y+H_0\sigma_2E_x-\dfrac{\partial\theta}{\partial x}\\[2ex] \rho\dfrac{\partial v_y}{\partial t}+H_0^2\sigma_2 v_x+H_0^2\sigma_1 v_y=H_0\sigma_2E_y-H_0\sigma_1E_x-\dfrac{\partial\theta}{\partial y}\end{cases} \tag{4.7}$$

From above equations, the equations for v_x and v_y, become respectively,

$$\begin{cases}\rho\dfrac{\partial^2 v_x}{\partial t^2}+2H_0^2\sigma_1\dfrac{\partial v_x}{\partial t}+\dfrac{(\sigma_1^2+\sigma_2^2)H_0^4}{\rho}v_x=f_1(E_x,E_y,\theta)\\[2ex] \rho\dfrac{\partial^2 v_y}{\partial t^2}+2H_0^2\sigma_2\dfrac{\partial v_y}{\partial t}+\dfrac{(\sigma_1^2+\sigma_2^2)H_0^4}{\rho}v_y=f_2(E_x,E_y,\theta)\end{cases} \tag{4.8}$$

where

$$f_1(E_x,E_y,\theta)=H_0\sigma_1\frac{\partial E_y}{\partial t}+H_0\sigma_2\frac{\partial E_x}{\partial t}-\frac{\partial^2\theta}{\partial t\partial x}+\frac{\sigma_2H_0^2}{\rho}$$

$$\times\left\{H_0\sigma_2E_y-H_0\sigma_1E_x-\frac{\partial\theta}{\partial y}\right\}+\frac{\sigma_1H_0^2}{\rho}\left\{H_0\sigma_1E_y+H_0\sigma_2E_x-\frac{\partial\theta}{\partial x}\right\}$$

$$f_2(E_x, E_y, \theta) = H_0\sigma_2 \frac{\partial E_y}{\partial t} - H_0\sigma_1 \frac{\partial E_x}{\partial t} - \frac{\partial^2\theta}{\partial t\partial y} - \frac{\sigma_2 H_0^2}{\rho}$$

$$\times\left\{H_0\sigma_1 E_y + H_0\sigma_2 E_x - \frac{\partial\theta}{\partial x}\right\} + \frac{\sigma_1 H_0^2}{\rho}\left\{H_0\sigma_2 E_y - H_0\sigma_1 E_x - \frac{\partial\theta}{\partial y}\right\} \qquad (4.9)$$

that is, these are eq. of forced oscillation due to the external force f_1 and f_2. Piddington disscussed the case in which external electric field and mechanical force along the one direction are applied.

Such a transient damping solution is interest in relation with the damped micro-pulsation which sometimes accompany with the s.c. of magnetic storm. Utashiro [19] and one of the author (Y. Kato) and others [20] pointed out from the observations with the induction-type magnetometer that there are damped micropulastion accompaying with s.c. of magnetic storm and in the hemisphere at daytime, the amplitude of the oscillation of dH/dt is greater than in that the hemisphere at night.

This fact is reverse to the statistical results obtained from the record of the variometer of H and D and also to that of rapid pulsation accompanying with bay disturbance. Fig. 2. shows some examples obtained with induction type magnetometer [21] at Onagawa.

These pulsations have the period of about 20~30 sec. and decay in time of several minutes.

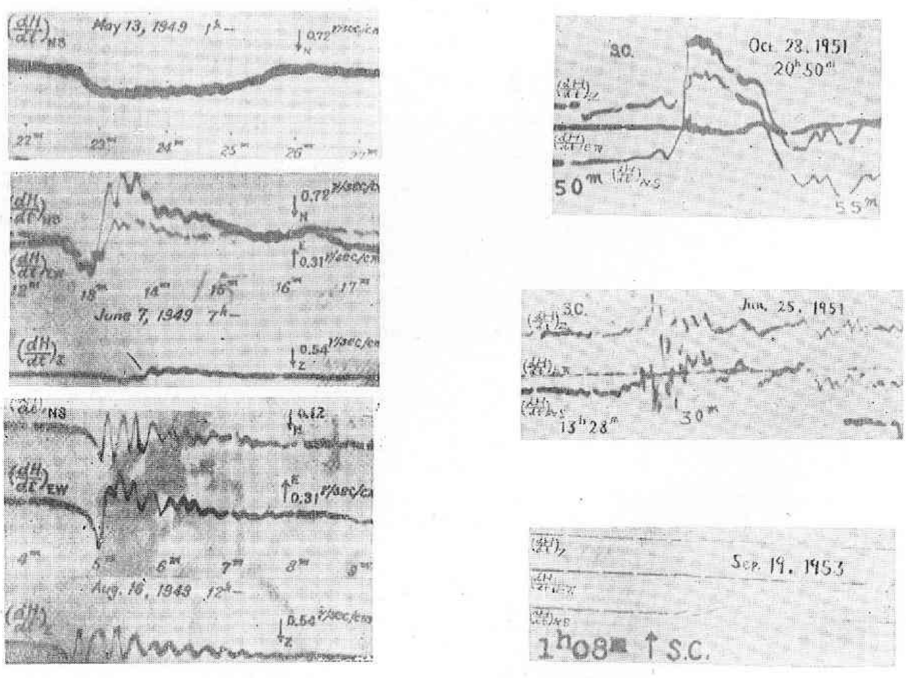


Fig. 2. Some examples of the damped pulsation accompanying the s.c. observed by induction-type magnetometer at Onagawa.

Considering the shielding effect of the ionosphere, and the fact that such short period disturbance is very remarkable in daytime hemisphere, we consider that its direct source lies in the lower part (e.g. E-layer) of the ionosphere. It will be likely to interprete that these pulsation is secondary transient phenomena due to sudden change in state of ionosphere which is set up by the extra-terrestrial agency showing the s.c. of magnetic storm. Such a sudden change in state may be accompanied by the sudden increase in electric current or sudden change of magnetic field etc. Since above stated oscillation in transient state which is solved as a particular solution of diff. eq. may be always followed in a sudden change in state occures, it may be considered that such a transient damped oscillation gives a possible explanation of above stated pulsation at s.c..

In the lower ionosphere, however, the ionic conductivities contribute appreciablly to the total one, so that the ionic motion must be studied by taking into account the collisional effects of the neutral particles in a transient state.

5 The aperiodic case

In this section, for simplicity, the case of $\lambda=0$ is discussed.

In eq. (3.1A), as Walén and Roberts discussed, function $S(z, t)$ is shown by the following Fourier integral

$$S_1(z,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\chi_1(K,t)\,e^{-iKz}dK \tag{5.1}$$

then, diff. equation which is satisfied for $\chi_1(K, t)$ is expressed by

$$\frac{\partial^2\chi_1}{\partial t^2}+\frac{K^2}{4\pi\sigma_3}(1-i\sigma)\frac{\partial\chi_1}{\partial t}+V^2K^2\chi_1=0 \tag{5.2}$$

Its solution is

$$\chi_1(K,t)=\chi_1(K,0)\,e^{-\frac{K^2t}{8\pi\sigma_3}+iK^2\frac{\sigma t}{8\pi\sigma_3}}\Big\{A_1e^{iKV\sqrt{\left[1-\left\{\frac{K(1-i\sigma)}{8\pi\sigma_3V}\right\}^2\right]}t}$$

$$+B_1e^{-iKV\sqrt{\left[1-\left\{\frac{K(1-i\sigma)}{8\pi\sigma_3V}\right\}^2\right]}t} \tag{5.3}$$

where $\chi_1(K, 0)$ is specified at the initial time and A_1, B_1 are constant. Therefore

$$S_1(z,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\chi_1(K,0)\,e^{-\frac{1-i\sigma}{8\pi\sigma_3}K^2t-iKz}\Big\{A_1e^{iKV\sqrt{1-\left\{\frac{K(1-i\sigma)}{8\pi\sigma_3V}\right\}^2}t}$$

$$+B_1e^{-iKV\sqrt{1-\left\{\frac{K(1-i\sigma)}{8\pi\sigma_3V}\right\}^2}t}\Big\}dK \tag{5.4}$$

Assuming that no higher harmonics of K are present at the initial state, $S_1(z, t)$ becomes

$$S_1(z,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\chi_1(K,0)\,e^{-\frac{1-i\sigma}{8\pi\sigma_3}K^2t-iKz}\left\{A_1e^{iKVt}+B_1e^{-iKVt}\right\}dK \tag{5.5}$$

while

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{1-i\sigma}{8\pi\sigma_3}K^2t-iKz\right\}e^{iKVt}dK=\sqrt{\left\{\frac{\sigma_3}{\pi(1-i\sigma)t}\right\}}$$

$$\times\exp\left[-\frac{2\pi\sigma_3(z-Vt)^2}{(1-i\sigma)t}\right] \tag{5.6}$$

and

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{1-i\sigma}{8\pi\sigma_3}K^2t-iKz\right\}e^{-iKVt}dK=\sqrt{\left\{\frac{\sigma_3}{\pi(1-i\sigma)t}\right\}}$$

$$\times\exp\left[-\frac{2\pi\sigma_3(z+Vt)^2}{(1-i\sigma)t}\right] \tag{5.7}$$

Let

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\chi_1(K,0)\,e^{-iKz}dK=\chi_{1,0}(z) \tag{5.8}$$

then, due to the Faltung theorem in Fourier transform $S_1(z,t)$ becomes

$$S_1(z,t)=\frac{1}{\pi}\sqrt{\frac{\sigma_3}{2(1-i\sigma)t}}\int_{-\infty}^{\infty}\chi_{1,0}(\xi)\left\{A_1\exp\left[-\frac{2\pi\sigma_3(z-\xi-Vt)^2}{(1-i\sigma)t}\right]\right.$$

$$\left.+B_1\exp\left[-\frac{2\pi\sigma_3(z-\xi+Vt)^2}{(1-i\sigma)t}\right]\right\}d\xi \tag{5.9}$$

$$=\frac{1}{\pi}\sqrt{\frac{(\sigma_3\,1+i\sigma)}{2(1+\sigma^2)t}}\left\{\int_{-\infty}^{\infty}\chi_{1,0}(\xi)\left\{A_1\exp\left[-\frac{2\pi\sigma_3(z-\xi-Vt)^2}{(1+\sigma^2)t}\right]\right.\right.$$

$$\left.+B_1\exp\left[-\frac{2\pi\sigma_3(z-\xi+Vt)^2}{(1+\sigma^2)t}\right]\right\}d\xi$$

$$+\int_{-\infty}^{\infty}\chi_{1,0}(\xi')\left\{A_1\exp\left[-i\frac{2\pi\sigma\sigma_3(z-\xi'-Vt)^2}{(1+\sigma^2)t}\right]\right.$$

$$\left.\left.+B_1\exp\left[-i\frac{2\pi\sigma\sigma_3(z-\xi'+Vt)^2}{(1+\sigma^2)t}\right]\right\}d\xi'\right\} \tag{5.10}.$$

Transforming the variables of integration

$$\xi=z\mp Vt+\eta\sqrt{\left(\frac{1+\sigma^2}{2\pi_3}t\right)}$$

$$\xi'=z\mp Vt+\eta'\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma\sigma_3}t\right)}$$

then,

$$S_1(z,t)=\frac{1}{2\pi}\sqrt{\left(\frac{1+i\sigma}{\pi}\right)}\int_{-\infty}^{\infty}\left[A_1\chi_{1,0}\left\{z-Vt+\eta\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma_3}t\right)}\right\}\right.$$

$$\left.+B_1\chi_{1,0}\left\{z+Vt+\eta\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma_3}t\right)}\right\}\right]e^{-\eta^2}d\eta$$

$$+\frac{1}{2\pi}\sqrt{\left(\frac{1+i\sigma}{\pi\sigma}\right)}\int_{-\infty}^{\infty}\left[A_1\chi_{1,0}\left\{z-Vt+\eta'\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma\sigma_3}t\right)}\right\}\right.$$

$$\left.+B_1\chi_{1,0}\left\{z+Vt+\eta'\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma\sigma_3}t\right)}\right\}\right]e^{-i\eta'^2}d\eta' \tag{5.11}.$$

Similarly, $S(z,t)$ which is reduced from eq. (3. 1B) may be obtained by reversing the sign of i involving eq. (5.11)

$$S_2(z,t)=\frac{1}{2\pi}\sqrt{\left(\frac{1-i\sigma}{\pi}\right)}\int_{-\infty}^{\infty}\left[A_2\chi_{2,0}\left\{z-Vt+\eta\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma_3}t\right)}\right\}\right.$$

$$+B_2\chi_{2,0}\left\{z+Vt+\eta\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma_3}t\right)}\right\}\Big]\,e^{-\eta^2}d\eta$$

$$+\frac{1}{2\pi}\sqrt{\left(\frac{1-i\sigma}{\pi\sigma}\right)}\int_{-\infty}^{\infty}\Big[A_2\chi_{2,0}\left\{z-Vt+\eta'\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma\sigma_3}t\right)}\right\}$$

$$+B_2\chi_{2,0}\left\{z+Vt+\eta'\sqrt{\left(\frac{1+\sigma^2}{2\pi\sigma\sigma_3}t\right)}\right\}\Big]\,e^{i\eta'^2}d\eta' \qquad (5.12)\,.$$

In the first approximation, these solutions show the distrubances traveling in $\pm$ z-directions with the velocity of $\pm V$.

An initial disturbance therefore splits into eight parts, everyone of which travels with the same velocity V. In eq. (5.11) and (5.12) respectively, the first two terms are similar corresponding one which was obtained by Roberts in the case of scalar conductivity. The other two terms which involve complex quantity in integrand represent the small disturbances which superpose upon the main disturbances represented in the first two terms respectively. Next, we will be to obtain the solutions of eq. (3.1A) and (3.1B) with operational method. Since these equations are similar to that discucssed by Roberts except that the coefficient is different, the similar reduction may be possible. In eq. (3.1A), considering the following transformation

$$t'=\left(\frac{\omega_0}{1+\sigma^2}+i\,\frac{\sigma\omega_0}{1+\sigma^2}\right)t \qquad (5.13)$$

and

$$z'=\left(\frac{1}{1+\sigma^2}\,\frac{\omega_0}{V}+i\,\frac{\sigma}{1+\sigma^2}\,\frac{\omega_0}{V}\right)z \qquad (5.14)$$

and set

$$S_1(z',t')=e^{-t'}Z(z',t') \qquad (5.15)$$

then, eq. (3.1A) becomes

$$\left(\frac{\partial}{\partial t'}-1\right)^2 Z(z',t')=\frac{\partial^3 Z(z',t')}{\partial t'\,\partial z'^2} \qquad (3.16)\,.$$

The above equation is identical with the Roberts one. Therefore, it may be possible to use the solution which was obtained by Roberts as the one of the present case. Similarly, transform

$$t'=\left(\frac{\omega_0}{1+\sigma^2}-i\,\frac{\sigma\omega_0}{1+\sigma^2}\right)t \qquad (5.17)$$

and

$$z'=\left(\frac{1}{1+\sigma^2}\,\frac{\omega_0}{V}-i\,\frac{\sigma}{1+\sigma^2}\,\frac{\omega_0}{V}\right)z \qquad (5.18)$$

and set $S_2(z',t')$ to the same form of eq. (5.15), then, eq. (5.16) is obtained again. While there are following relations

$$\frac{\sigma\omega_0}{1+\sigma^2}=\frac{\sigma_2H_0^2}{\rho}\,,\qquad \frac{\omega_0}{1+\sigma^2}=\frac{\sigma_1H_0^2}{\rho}\,,\qquad \frac{1}{1+\sigma^2}\,\frac{\omega_0}{V}=\frac{1}{V}\,\frac{\sigma_1H_0^2}{\rho}\,,$$

$$\frac{\sigma}{1+\sigma^2}\,\frac{\omega_0}{V}=\frac{1}{V}\,\frac{\sigma_2H_0^2}{\rho}\,,$$

eq. (5. 13), (5.14), (5.17) and (5.18) become

$$t' = \left(\frac{\sigma_1 H_0^2}{\rho} \pm i\frac{\sigma_2 H_0^2}{\rho}\right)t \tag{5.19}$$

and

$$z' = \left(\frac{\sigma_1 H_0^2}{\rho} \pm i\frac{\sigma_2 H_0^2}{\rho}\right)\frac{z}{V} \tag{5.20}$$

respectively.

Useing the above relations, eq. (5.15) reduce to

$$S_{1,2}(z, t) = \exp\left[-\frac{\sigma_1 H_0^2}{\rho}t \mp i\frac{\sigma_2 H_0^2}{\rho}t\right] Z(z', t') \tag{5.21}$$

that is, this representation is identical with the damped oscillation obtained in §. *4*. Thus, in transient state following the initial state, it may be expected that there is a damped oscilltaion of this kind. This matter is very remarkable difference with compared to the case of scalar conductivity or $\sigma_2/\sigma_1 \ll 1$. That is, in the later case when we have boundary value problem, there are free mode solutions of poloidal and toroidal type which has enumerable infinite frequencies independently each other and any decaying field may be constructed with superposition of these free modes, while in the case of $\sigma_2/\sigma_1 > 1$, there is a damped oscillating solution with the period which is determined by physical constants of the medium even in the case in which the medium is unbounded. For the binary gas, this frequency is identical with the gyro-frequency of the heavy ion which has spiral motion about a magnetic line of force [18]. In the case of scalar conductivity, in order to persist the particular free mode, we must have the special exciting process, while the present damped oscillation may be always occured at the change of state.

According to Roberts, diff. eq. which is satisfied with function $Z(z', t')$ reduces to a simpler form operationally.

Operate $pe^{-pt'}$ to the both sides of eq. (5.16) and at $t'=0$, initial conditions are specified as follows

$$Z = Z_0(z'), \qquad \frac{\partial Z}{\partial t'} = Z_1(z') \tag{5.22}$$

then, diff. eq. (5. 16) becomes

$$\frac{\partial^2 \bar{Z}}{\partial z'^2} - \mu^2 \bar{Z} = g(z') \tag{5.23}$$

where

$$\mu = \sqrt{p} - \frac{1}{\sqrt{p}} \tag{5.24}$$

$$g(z') = Z''_0(z') - Z_1(z') - (p-2) Z_0(z') \tag{5.25}$$

and

$$\bar{Z}(z', p) = p\int_0^\infty Z(z', t') e^{-pt'} dt' \tag{5.26}$$

The general solution of eq. (5.23) consist of two parts, the one represents an effect of sudden disturbing source (complementary integral of diff. eq.) and the other represents an effect of free decay of initial configuration (particular integral of diff. e.q.)

The following representation of complementary integral was shown by Roberts

$$S(z', t') = \int_0^{t'} f(t' - \tau)\, \Phi(z', \tau)\, d\tau \tag{5.26}$$

where

$$S(0, t') = f(t') \tag{5.27}$$

and

$$\Phi(z', t') = \frac{e^{-t'}}{2\sqrt{(\pi t'^3)}} \int_0^\infty \left[\frac{(x+z')^2}{2t'} - 1\right] e^{-(x+z')^2/4t'}\, I_0\left[2\sqrt{(xz')}\right] dx \tag{5.28}$$

$I_0(x)$ is the modified Bessel function of the first kind of zero order. When the sudden disturbance is Heavisides unit function $1\ (t)$ or Diracs delta function $\delta\ (t)$ type, then $S(z, t)$ becomes

$$S(z', t') = \int_0^{t'} \frac{1}{\delta} \begin{Bmatrix} (t'-\tau) \\ (t'-\tau) \end{Bmatrix} \Phi(z', \tau)\, d\tau \tag{5.29}$$

6 Boundary conditions and Stationary State

In fact, since the observations are made outside of conducting, we must require the solutions in the exterior and interior of conducting which are satisfied with boundary conditions. Though above problem must be treated that of reflection and refraction of waves strictly, the quasi-stationary method may be possible even in the dielectrics, when the rate of time change is slow comparatively. In this mater, electromagnetic fields must be considered as resulting fields of poloidal and toroidal types in the interior of conducting as well as in the exterior. In the later, however, due to the factor λ^2 toroidal field will be small compared with poloidal one.

At the boundary, being satisfied the condition of continuity in electric and magnetic field, then v_z is not in general vanish at the boundary, that is, boundary surface is removed. It may be, however, expected that due to the factor λ^2, v_z is small compared with the velocities in lateral direction

Moreover, for time depending solutions, j_z is not vanish at the boundary, but its order of magnitude equals to displacement current which was neglected at the beginning.

In transient case, the amplitude of disturbance for exterior region may be in principle obtained with calculating the coefficients of time dependence which is resulting from boundary conditions operationally, in the present case, however, it is almost impossible since due to the coupling between poloidal and toroidal fields those coefficient become remarkable complexity. Even for the periodic case, there are eight unknown coefficients in the solution satisfied with conducting, therefore, the matter becomes troublesome. Also, in real gas, it is unlikely that there is such a sharp discontinuity of gas parameters.

When the extenral disturbance is such as Heaviside unit function type, after the sufficient long time, transient disturbance will decay and the state becomes stationary. For such a case,

$$\boldsymbol{h}^{(s)}=\begin{cases}\dfrac{\partial P}{\partial x}\dfrac{dS}{dz}\\[2mm] \dfrac{\partial P}{\partial y}\dfrac{dS}{\partial z}\\[2mm] \lambda^2 PS\end{cases} \quad (6.1) \qquad \boldsymbol{h}^{(t)}=\begin{cases}\dfrac{\partial P}{\partial y}T\\[2mm] -\dfrac{\partial P}{\partial x}T\\[2mm] 0\end{cases} \quad (6.2)$$

$$\boldsymbol{E}^{(s)}=0 \quad (6.3) \qquad \boldsymbol{E}^{(t)}=\begin{cases}\dfrac{\lambda^2}{4\pi\sigma_0}\dfrac{\partial P}{\partial x}T'\\[2mm] \dfrac{\lambda^2}{4\pi\sigma_0}\dfrac{\partial P}{\partial y}T'\\[2mm] \dfrac{\lambda^2}{4\pi\sigma_0}PT\end{cases} \quad (6.4)$$

where

$$\frac{dT'}{dz}=T(z) \qquad (6.5)\,,$$

the electrie field $\boldsymbol{E}^{(t)}$ represented by eq. (6.4) is the field due to static porlarization, and

$$\theta^{(s)}=-(4\pi)^{-1}\lambda^2 H_0 PS \quad (6.6) \qquad \theta^{(t)}=0 \quad (6.7)$$

$$v^{(s)}=\frac{\lambda^2}{4\pi H_0(\sigma_1^2+\sigma_2^2)}\begin{cases}\sigma_1 S\dfrac{\partial P}{\partial x}\\[2mm] \sigma_1 S\dfrac{\partial P}{\partial y}\\[2mm] \lambda^2\sigma_1 SP\end{cases} \qquad (6.8)$$

$$v^{(t)}=\begin{cases}\dfrac{\lambda^2}{4\pi H_0(\sigma_1^2+\sigma_2^2)}\dfrac{\partial P}{\partial y}\left(\dfrac{\sigma_1^2+\sigma_2^2}{\sigma_0}T'+\sigma_2 S\right)\\[2mm] -\dfrac{\lambda^2}{4\pi H_0(\sigma_1^2+\sigma_2^2)}\dfrac{\partial P}{\partial x}\left(\dfrac{\sigma_1^2+\sigma_2^2}{\sigma_0}T'+\sigma_2 S\right)\\[2mm] 0\end{cases} \qquad (6.9).$$

Substitute above expressions to eq. (2.8), (2.9) and (2.10), then $S(z,t)$ and $T(z,t)$ become

$$\begin{cases}S(z)=az+b\\ T(z)=\text{const}=0\end{cases} \qquad (6.10)\,.$$

In general, therefore, the electric current is not vanish, that is, after the sufficient long time, the static shielding are remained. The static porlarization field (6.4) are cancealed by induction field $[\boldsymbol{v}+\boldsymbol{H}]$ but due to the pressure gradient effect the total electric field $\boldsymbol{E}'$ is not vanish. Such a results are different in the case of scalar conductivity and also of the solid induction. In the later, for poloidal type, the current decays completely and is not shielding and for toroidal, there is static shielding due to porlarization.

Omitting the pressure gradient in the eq. of motion, then

$$\left\{\begin{array}{l}\frac{\partial^2 S}{\partial z^2} - \lambda^2 S = 0 \\ T = \text{const} = 0\end{array}\right. \tag{6.11}$$

that is, the current component in lateral direction vanish.

This result is accordance with Piddington's one, that is, in order to present a stationary current there is required mechanical force.

7 Conclusion

Assuming the simple model, we discussed the disturbance in an incompressible conducting fluid, in particular, the damped periodic disturbance.

The macroscopic relations and quantities which were used in the above discussions must be reexamined in the ionoshere.

Taking into account the collisional effects of background material, the dispersion equation (3.5) approximately valids in a frequency range in which ω is smaller than 1c/sec in the E region.

In this case, ρ is the density of the neutral particle, in consequsnce, the second term becomes much larger than the first in the denominator, then the character of the two polarized waves becom very different even in the case in which ω is relatively low-frequency.

References

1 SUGIURA, M.: Electromagnetiic induction in the ionosphere *Jour. Ionosphere. Res. Jap* **3**, 65–72, (1949).
———— The shielding effect of the ionosphere. ibid. **4**, 31–36, (1950).

2 OGUTI, T. and NAGATA, T.: Model experiments of the shielding effect of the ionosphere *ibid*. **8**, 171–183, (1954).

3 ASHOUR, A. A. and PRICE, A. T.: The induction of electric currents in a non-uniform ionosphere. *Pro. Roy. Soc.* A**195**, 198–224, (1948).

4 ROBERTS, P. H.: Hydromagnetic disturbances in a fluid of finite conductivity *Ap. J* **122**, 315-326, (1953).

5 HINES, C. O.: Generalized magneto-hydrodynamic formulae *Proc. Camb. Phil. Soc.* **49**, 229–307, (1953).

6 DUNGEY, J, W.: The propagation of Alfvén waves through the ionosphere. *Ionosph. Res. Sci. Rep. Pennsylvania State Univ.* Feb., **15**, No. 57, (1954).

7 KATO, Y. and WATANABE, T.: A possible explanation of the cause of giant pulsations. *Sci. Rep. Tohoku Univ. Ser.*, 5, *Geophys*. **6**, 95–103, (1955).

8 HINES, C. O.: Hydromagnetic resonance in ionospheric waves. *J. A. T. P.* **7**, 14–30, (1955).

9 KATO, Y. and AKASOFU, S.: Outer atmospheric oscillation and geomagnetic micropulsation. *Sci. Rep. Tohoku Uni., Ser.* 5, *Gepohys.*, **7**, 103–123, (1955).

10 COWLING, T. G.: The electrical conductivity of an ionised gas in the presence of a magnetic field. *M. N. R. A. S.* **33**, 90–98, (1933).

11 WALEN, C. C.: On the theory of sun-spots. *Ark. f. Mat. Astr. o Fys.* **30A**, No. 15, (1944).

12 PIDDINGTON, J. H.: Electromagnetic field equations for a moving fluid with Hall conductivity. *M. N. R. A. S.* **114**, 638–650, (1954).

13 PRIÉCE, A. T.: Electromagnetic induction in a semi-infinite conductor with a plane boundary. *Quart. Jour. Mech. and Apply. Math.* **3**, 385–410, (1950).

14 GORDON, A. N.: Electromagnetic induction in a uniform semi-infinite conductor. *ibid* **4**, 106–128, (1951).

15 ALFVÉN, H.: "Cosmical Electrodynamics", (1950), (Oxford).

16 STRATTON, J. A.: "electromagnetic Theory" p. 393, (1941), (Mcgraw-Hil).
17 DUNGEY, J. W.: The attenuation of Alfvén waves. Jour. Geophys. Res. **59**, 323-328, (1954).
18 PIDDINGTON, J. H.: The motin of ionized gas in combined magnetic, electric and mechanical fields of force. *M. N. R. A. S.* **114**, 651-663, (1954).
19 UTASHIRO, S.: On the character of the micropulsation in the magnetic storm. *Jour. Geomag. and Geoelect.* **1**, 59-66, (1949).
20 KATO, Y. and OSSAKA,: J. and OKUDA, M. Investigation on the magnetic disturbance. by induction magnetograph. *Sci. Rep. Tohoku Univ. Ser.* 5, *Geophys.* **6**, 85-122, (1953)
21 GRENET, G., KATO, Y.,——, ——.: Pulsations in terrestrial magentic field at the time of bay-disturbance. *ibid.* **6**, 1-10, (1954).